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A New Method of Determining H by Means of the Discharge from the Wehnelt Cathode

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A NEW METHOD OF DETERMINING H BY MEANS OF THE DISCHARGE FROM THE WEHNELT CATHODE

BY

LARS ALVIN WELO
B. S. North Dakota Agricultural College, 1911

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

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IN PHYSICS

IN

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May 21 1905

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

LARS ALVIN WELO

ENTITLED A NEW METHOD OF DETERMINING H BY MEANS OF THE DISCHARGE FROM THE WEHNELT CATHODE

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE IN PHYSICS

Chas. T. Knipp
In Charge of Major Work

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Head of Department

Recommendation concurred in:

Committee
on
Final Examination

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I INTRODUCTION

A common method of determining the ratio of the charge to the mass of an electron is to use a cathode tube and to measure the deflection of the beam of particles by a magnetic field; the other necessary relation being obtained, either by observing the fall of potential at the cathode, or the deflection of the rays as they pass between the plates of a condenser.

The fact that the measurement of e/m with a cathode tube involves the deflection produced by a magnetic field, suggests that we have here a method of measuring magnetic fields, provided we assume the ratio e/m. For low velocities of the electron this ratio, 1.765×10^7, is constant and is well determined by measurements on the cathode rays and more especially on the Zeeman phenomenon. This investigation, therefore, concerns itself with the development of such a method for determining the horizontal component of the earth's magnetic field.

Since this field to be measured is always very weak compared with those employed in the determination of e/m, one must choose a form of tube which will give a measurable deflection, yet which will not be inconveniently long. Tubes equipped with cold cathodes such as the Braun tube impart high velocities of the order 9×10^9 cm/sec. to the electrons. The Wehnelt hot lime cathode, on the other hand, because of the lower potential differences which can be used, give velocities of about 1.6×10^9 cm/sec. and since the magnetic deflection is inversely proportional to the velocity of the electron, it is obvious that the Wehnelt cathode is advantageous. An additional advantage in using this cathode is that the active part can be limited to a very small piece of lime placed on the heated platinum strip. With

a good charcoal liquid air vacuum one can thus get a well defined beam of long range without the use of screens and diaphragms.

II THE THEORY

Development of Formula.- Figure I is a sketch of the tube used. The anode is shown at (A) and the cathode (C) at a distance (l) above the screen S. The electrostatic plates of length (d) and of distance apart (t) are shown with their most remote end at a distance (l') from the screen. Calling y the total electrostatic deflection, y_{mn} the deflection after the rays have just passed the plates, and y' the additional deflection while on their way to the screen, we have[1], Y being the electric force perpendicular to the poles and V the velocity of the electrons downward

$$y = y_{mn} + y' = \frac{1}{2}\frac{Ye}{m}\frac{d^2}{v^2} + \frac{Ye}{m} \cdot \frac{d}{v} \cdot \frac{l'-d}{v}$$

$$= \frac{Ye}{mv^2} d\left(l'-\frac{d}{2}\right)$$

putting $d\left(l'-\frac{d}{2}\right) = A$

$$y = AY\frac{e}{mv^2} \qquad (1)$$

Consider the circle of diameter d and in position shown, as the boundary of a uniform magnetic field perpendicular to the plane of the paper, and writing z, z_{mn} and z' for y, y_{mn}, y' respectively, we have for the magnetic deflection

$$z = z_{mn} + z' = \frac{1}{2}\frac{He}{mv}d^2 + \frac{Hed}{m} \cdot \frac{l'-d}{v}$$

But in this case the earth's field is effective all the way to the screen so the second term is zero since d becomes equal to l, and we have

$$z = \frac{1}{2}\frac{He}{mv}l^2$$

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[1] Thomson, Conduction of Electricity Through Gases, 2 Ed.,p.117

[illegible]

[illegible]

[illegible]

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$$[illegible]$$

[illegible]

$$[illegible]$$

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$$[illegible]$$

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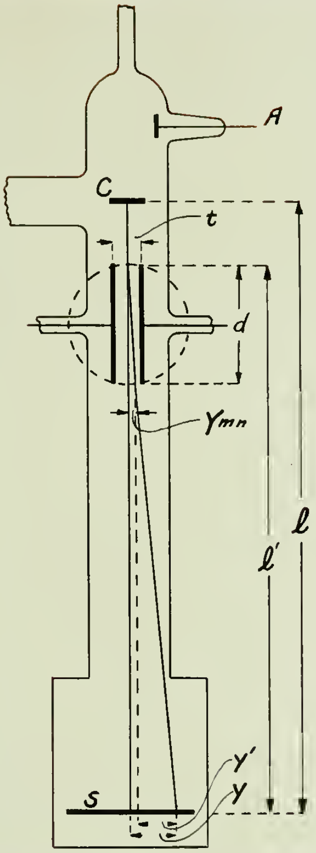

FIG. I

Putting

$$B = \frac{l^2}{2}$$

$$z = B\frac{He}{mv} \qquad (2)$$

Now eliminating v between equations (1) and (2) and solving for H^2 we have

$$H^2 = \frac{AY}{y}\frac{m}{e}\frac{z^2}{B^2}$$

$$= \frac{A}{B^2}\frac{m}{e}\frac{Yz^2}{y}.$$

Putting

$$\frac{H}{B^2}\frac{m}{e} = C = \text{a constant}$$

$$H^2 = CY\frac{z^2}{y}$$

$$H = Dz\sqrt{\frac{Y}{y}} \quad \text{where } D = \sqrt{C}$$

But $Y = \frac{V \times 10^8}{t}$ where V is the electromotive force between the plates, measured in practical units, and t their distance apart,

Hence

$$H = Dz\sqrt{\frac{V \times 10^8}{ty}}$$

$$= \frac{10^4 D}{\sqrt{t}} z\sqrt{\frac{V}{y}}$$

$$= Ez\sqrt{\frac{V}{y}} \quad \text{where}$$

$$E = \frac{10^4 D}{\sqrt{t}}$$

Reduction of the Photographs.- It is obvious that the earth's magnetic field cannot be eliminated for a zero reading of the deflection. To measure this deflection, then, it was found necessary to place the tube in a frame so that it could be turned about a vertical axis. It is apparent from Fig. 1, since the cathode beam is always deflected in the same direction, that a circle will be described on a photographic plate at S if the plate turns with the tube, the

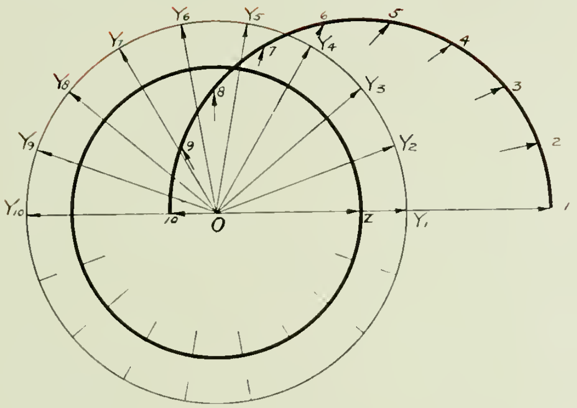

FIG. II

radius of which being the deflection sought. What one gets when both the electrostatic and magnetic fields act on the beam is not so apparent; however, the resulting trace on the photographic plate is easily deduced graphically on the assumption that the plate does not turn with the tube, and then extending the discussion to the real case where it does.

In Fig. 2, O is the point on the plate, but not necessarily in the axis of the tube, where the beam would strike if neither field were active. The vector Oz is the magnetic deflection and O_y the electrostatic deflection, it being assumed different and larger than Oz for convenience.

Imagine for a moment that the plate does move with the tube and that only the magnetic field is active. The heavy circle of radius Oz is described. Now, fix the plate and charge the electrostatic plates which are in such a position that they would produce the deflection Oy acting alone. With both fields acting for this position the two deflections would be added and the beam would fall on the point 1 which is one point on the desired curve. Now, turn the tube, say 20°. The magnetic deflection has the same direction and magnitude as before but the electrostatic plates have also been turned through an angle of 20° and their deflection alone would be OY_2. The actual deflection is the vector sum of O_z and OY_2 which brings the beam to 2. Turning another 20° the electrostatic deflection is OY_3 which added to O_z gives the point 3. The points 4, 5 .... 10 were similarly obtained and by continuing, or turning as before, but in the opposite direction, we would get points below the line 1 - 10. It was found that these points were on the circumference of a circle of radius OY and center z. If our plate were stationary then we would have

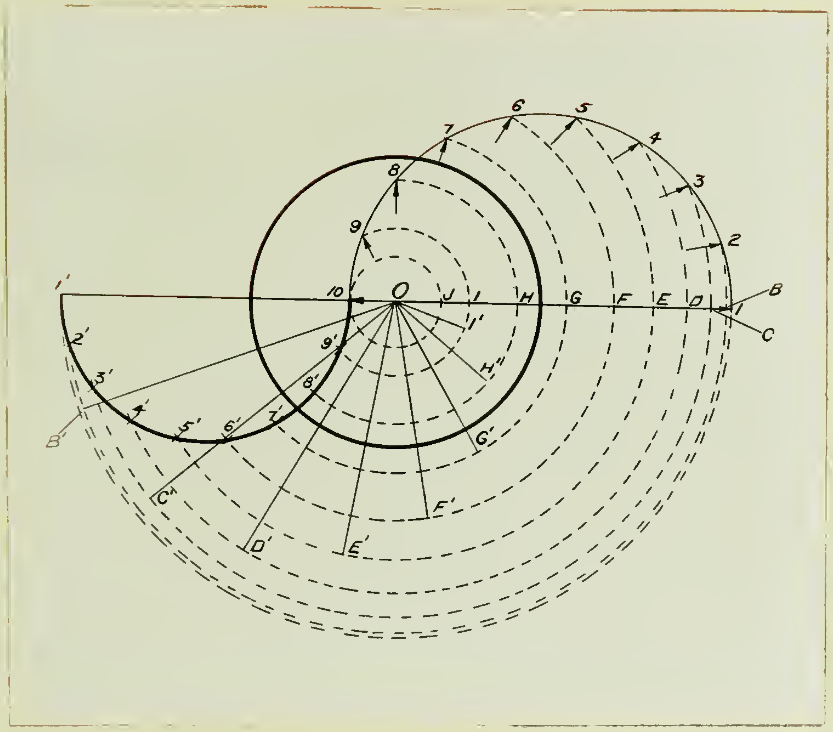

FIG. III

two circles after turning twice, once with the magnetic field active, and once with both fields acting, the radii being the magnetic and electrostatic deflections.

In the apparatus used the photographic plate turned with the tube and the deduction of results is made from Fig. 3.

The heavy complete circle has the same meaning as in Fig. 2 as has the circle 1, 2, 3,........10. We are to deduce the figure described for half a revolution of the tube. If the plate were stationary we would get the point 1, but at the end of half a revolution the point 1 would be at 1' on the other side of 0, hence 1' is one point on the curve desired. After turning 20° with stationary plate we should have the point 2, but this would after half a turn be some where on the opposite side of 0. The point is located by describing a circle (dotted) through 2 with 0 as center and the point is somewhere on its circumference. Take the chord B2 and remembering that 01 has turned through 20°, we must apply the chord B2 at the point B', giving 2' as the second point on the curve desired. After turning another 20° we would have the point 3 with stationary plate. Draw the circle (dotted) through 3 with 0 as center. Apply the chord C3 from C' because of the additional rotation and we get the third point 3'. Drawing circles through 4, 5, 6 ....10 about 0 and applying chords D_4, E_5 .... J_{10} to the points D', E'.....J' we get the points 4', 5'.......10'. The points 1', 2' ......10' are found to be on the circumference of a circle of radius O_z but its center is distant from the center of the circle described by a magnetic field alone by O_y, the electrostatic deflection. The radius of either circle is the magnetic deflection and the distance between their centers or corresponding edges of circles, if measured in a direction parallel to the

line joining the centers, is twice the electrostatic deflection.

It was found early during the course of the work that there was little hope of securing a permanent electrostatic deflection. As soon as the plates were charged the beam would jump in a plane perpendicular to the plane of the plates, a distance proportional to the potential difference applied but it would not stay there. It would creep back to the zero position, first rapidly and then more slowly. Similar difficulties have been observed in cathode tubes by Thomson[1] Wehnelt[2], Milham[3] and others. Thomson traced the effect to the conductivity imparted to the residual gas by the passing cathode rays. He measured the conductivity of the gas and found that it decreased with improved vacuum so that if a high enough vacuum was reached the beam would stay permanently deflected. The positive and negative ions between the plates diffuse until the positive plate becomes coated with negative ions and the negative with positive ions, with the result that the electric intensity gradually vanishes. The Wehnelt cathode ceases to act if the pressure is less than about 0.0008 mm. and no permanent deflection could be obtained within the range of pressures for which this cathode will work. Wehnelt has constructed a Braun tube equipped with this form of cathode and obtained a permanent deflection, but he used a diaphragm, which is impracticable here because the particles are deflected magnetically out of the axis of the tube as soon as they leave the cathode.

The work of Milham suggested that use might be made of the

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[1] J. J. Thomson, Phil. Mag., Vol. 206, 1897, pp. 293-316.

[2] Wehnelt, Phys. Zeit, 1905, pp. 732-733; Verh. d. Deutsch. Phys. Gesell., 1903, p. 29.

[3] Milham , Phys. Zeit., 1901, p. 637.

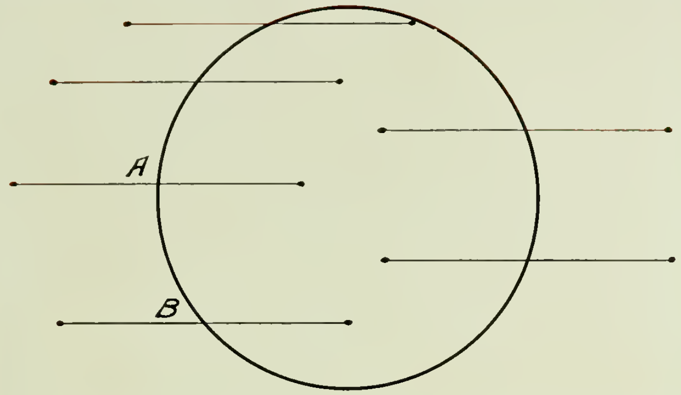

FIG. IV

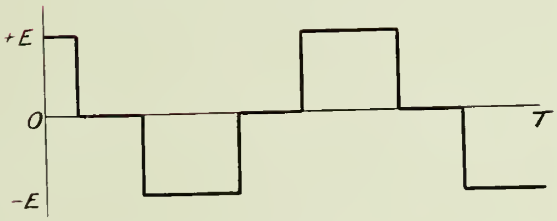

FIG. V

jump which he found to be proportional to the change in the potential difference between the plates. One jump, however, would hardly leave an impression on the photographic plate, so use was made of a commutator. The alternating electromotive force would cause the beam to vibrate across the plate as long as necessary to affect it. We should have a straight line traced out on the plate in a plane perpendicular to the plane of the plates and, referring to Fig. 4, we begin to expose the plate with the beam striking it at, say, A and applying the alternating electromotive force, resulting in the line crossing the circle at A. The ends of the line are more dense than the line itself because the curve of electromotive forces takes the form shown in Fig. 5. During intervals of zero potential the beam would not have time to creep appreciably from the position of maximum deflection.

Having made an exposure at A, we break the circuit and short the plates. Turning the tube, a part of the trace due to the magnetic deflection will be described until B is reached. The tube is stopped and an alternating electromotive force again applied. As many exposures as desired may be made for the electrostatic deflection on a given plate. The electrostatic deflection is half the distance between the spots.

III THE APPARATUS

Figure 6 is a sketch of the apparatus used. There is an outer wooden frame equipped with leveling screws so the tube can be made vertical. The tube is firmly fastened to the inner frame by clamping with a lining of tape and felt at T and by blocking by means of corks at G placed between the widened part of the tube and the vertical parts of the frame. Getting the tube and the frame in line

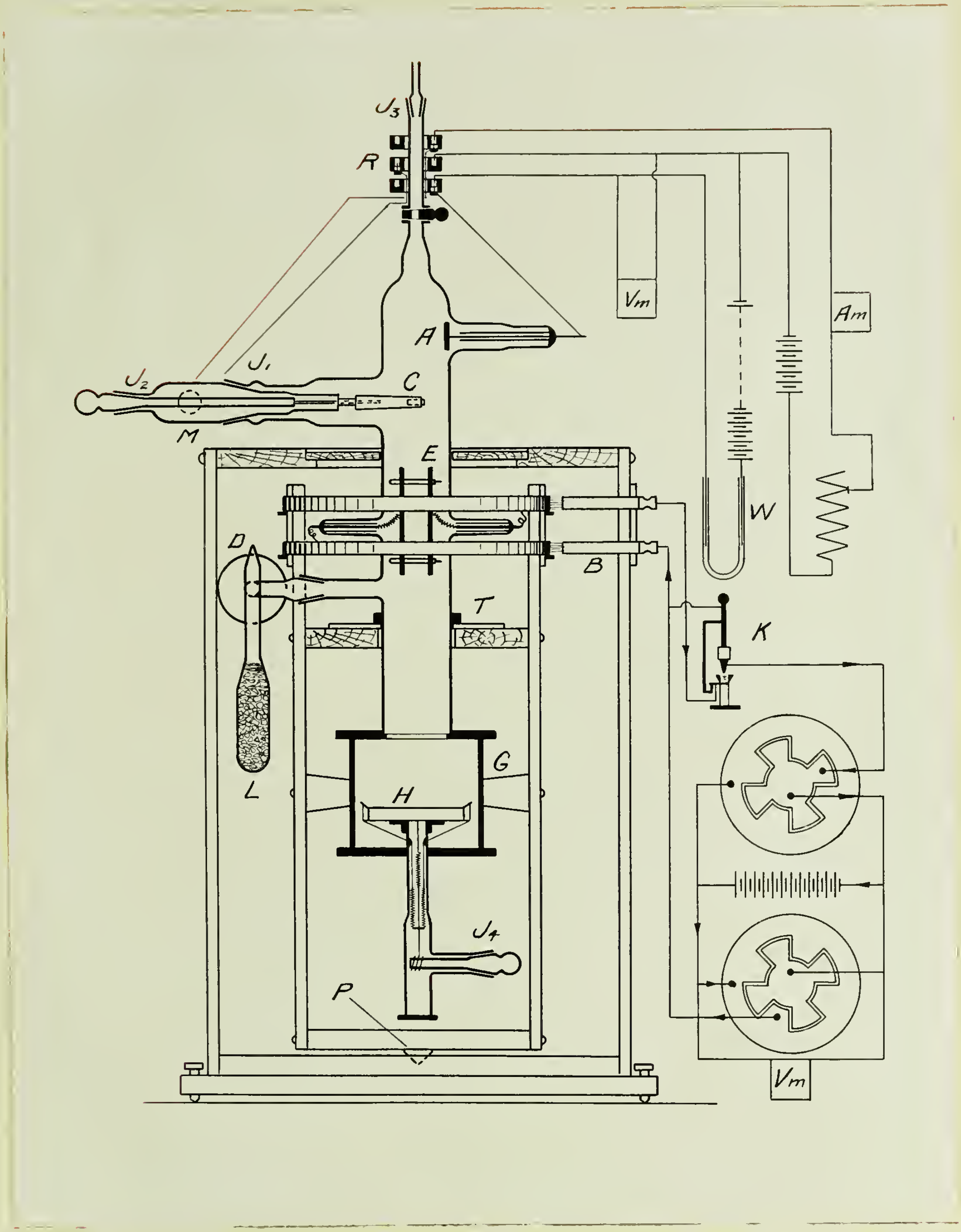

FIG. VI

is a matter of cutting the corks to the right length. At P the inner frame rests on a pivot supported at the bottom of the outer frame. The other bearing is the tube itself where it passes through the upper part of the outer frame at E. This opening is felt lined and adjustable.

The anode is at A and the cathode is indicated at C. The direction the beam will take is uncertain so the cathode must be made adjustable. J_1 is a ground joint and by turning here the beam can be shifted in a plane perpendicular to the plane of the drawing. It will be seen later that by turning the stem going through the joint J_2 the corresponding adjustment in the plane of the drawing can be made.

The plates which produce the electrostatic deflection are shown below E. They are of aluminum and are rigidly fastened to four hard rubber posts. By placing friction pins in one end of each of the posts these plates are held at any desired position in the discharge tubes.

Electrical contact is made with the plates by small coiled springs which are soldered to the electrodes fused into the ends of the two side tubes shown. These springs bend either up or down, permitting a considerable range of positions for the plates and assure at the same time, good contact by their pressure on the backs of the plates.

These electrodes are connected, one to each of the amalgamated copper rings shown, one above and one below the two side tubes. These rings are rigidly fastened to the frame work but insulated from it by hard rubber blocks and pins. They make contact with the brushes shown at B which lead to two binding posts and thence to the

commutator circuit.

Near the top of the apparatus at R are three hard rubber rings hollowed out so as to contain an annulus of mercury. They are fastened to the stem of the tube by split corks. The upper two are connected to wires which lead to the two electrodes that are set in the side tubes perpendicular to the plane of the drawing at the position indicated by the dotted circle at M. These wires carry the heating current for the platinum strip which constitutes the cathode on which the spot of lime is placed. One of these annular contacts serves also as the connection to the negative pole of the high potential battery producing the discharge. The lower ring serves as a connection, through the water resistance W, between the anode and the positive pole of the high potential battery.

D is a drying bulb containing P_2O_5, its stem being perpendicular to the plane of the drawing. L is a charcoal bulb. The bulbs are connected to a common stem thence by a ground joint to the discharge tube as shown. J_3 is a ground glass joint whose upper part leads to the Gaede capsule pump. The only special precaution which need be taken about this joint is that the tube must be perfectly aligned by adjusting the corks at the lower end of the tube. Otherwise the joint may be partly wedged apart and air admitted during the exposure.

Referring to the commutator circuit, the two wheels shown diagrammatically are fastened to the same axle so the reversals are synchronous. When the wheels are in the position shown, relative to the brushes indicated by the spots, the displacement current takes the direction indicated by the arrows provided the bar of the high insulation key K is down. When the bar of the key is up the commutator

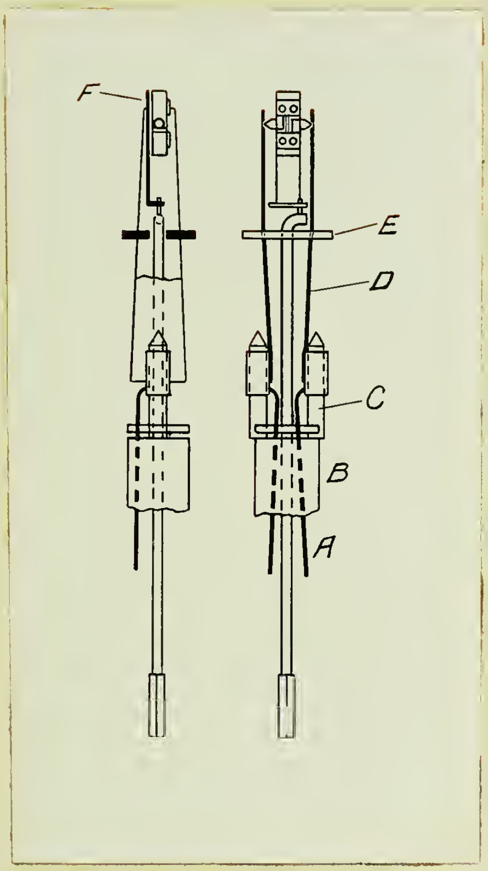

FIG. VII

circuit is broken and the side bar shorts the plates and no field exists.

Reference has been made to the adjustment of the beam in the plane of the paper. In Fig. 7 A is one of the leads leading to the electrode at M, Fig. 6. B is the end of the glass tube going through the joint J_1. At C are two glass bars fused to the ends of this tube. The two heavy vertical lines are spring brass strips soldered to the two split brass sleeves which fit snugly over the glass bars. E is a thick piece of mica which adds stiffness to the system and serves as a bearing for the central rod, the lower end of which fits into the glass stem going through J_2, Fig. 6. F is a brass strip and at the lower end has a slit cut crosswise, this part being at right angles to the main part of the strip. A pin in the bent part of the long central rod fits into this slot and as the stem through the joint J_2, Fig. 6, is turned, the central rod turns and the cathode proper, mounted on the strip F, is rocked through a considerable angle. The bearings of the cathode are two cone-shaped pins which fit into V-shaped dents in the brass strips. To insure good contact the strips are adjusted to press against the pins.

A mica strip is placed upon the brass strip F and two L-shaped blocks mounted upon it, making sure that they are perfectly insulated from each other and from the brass strip on which they are mounted. The circuit for the heating current is broken until the platinum strip is clamped between the blocks. The platinum strip should be thin and narrow so that small heating currents give high temperatures without the danger of cracking the glass at the sealed in platinum electrodes. A spot of lime 0.2 millimeters in diameter may be readily gotten so the strip need not be over 0.3 mm. wide.

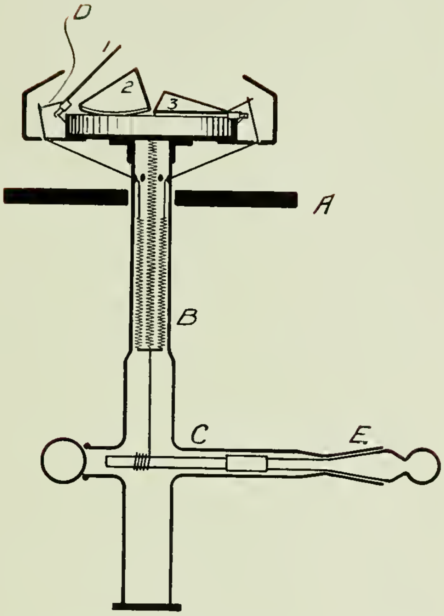

FIG. VIII

A special plate holder and shutter was devised. This is detailed in Fig. 8 where A, a thick glass plate closes the lower end of the tube. A hole is bored through this plate and a glass tube B with a cross-section C inserted and waxed on with Bank of England wax, which wax was used in fastening the wide part of the tube to the narrow part and, it may be remarked, was also used as a source of lime for the cathode. The holder itself is a short brass cylinder closed at one end about one centimeter deep and nine centimeters in diameter. A split ring fastened to its center side, secures it snugly to the end of the glass tube. The shutter was made by cutting a circular sheet of thin aluminum into six segments. The diameter of the sheet was a little larger than that of the plate holder so the rims of the segments might be bent down to prevent light from entering the holder edgewise and also so that when fitted on the holder, the edges of the segments would extend over about a centimeter. Each segment was hinged at its base as shown at 1 in the figure. Since the leaves lap over each other they must come up at different times and in the figure four of them are shown. Leaf 1 was the first to go up, 2 started next, then 3, and 4, which is seen edgewise, has not yet moved. The leaves are moved by a system of springs and strings. D is a small strip riveted to the leaf near its hinge and bent up a little, having a string fastened to its end. The string goes through a loop in a wire, here shown as a hole in the heavy wire soldered to the bottom of the holder and bent up and inward to stop the leaf from going so far that it cannot close of its own weight. The string next goes through small holes in the glass tube above the plate A and then to the upper end of a spring. There is one spring and one string for each of the six leaves or segments,

and the adjustment of the time of coming up of the leaf is made by shortening or lengthening the string. The springs are joined at the lower end and their weight is supported by the long control spring hung from the bottom of the holder. The junction at the lower end is fastened to a string which winds upon the stem passing through the ground joint E. The openings shown at the ball and plate at the bottom are convenient when making adjustments. They are closed with wax. The stem is flexible, being made in two parts joined by a tightly fitting coiled spring, and thus relieves all strain on the ground joint E.

IV MANIPULATION

When making a determination the pump was generally run for over an hour after having closed the bottom of the tube with the plate holder shown in Fig. 8. Half-and-half wax was used in making the joint air tight. The charcoal bulb was gently heated during the time of pumping in order to drive off any air absorbed at ordinary temperatures. The shutter of the plate holder was efficient enough to prevent fogging by the light from the flame and the partly darkened room. Having pumped for over an hour, the charcoal bulb was immersed in liquid air for about 15 minutes before making an exposure. A potential difference of 800 volts from a high potential battery was then applied and the resistance in the heating circuit adjusted until the platinum strip glowed a faint red. It was left this way for about 5 minutes. Then the heating current was gently increased until a barely visible beam appeared. The beam increased in intensity and in a minute or two would give a phosphorescent spot on the leaves of the plate holder which were covered with a thin layer of willemite.

The first adjustment is to shift the cathode so the spot

will stay on the plate holder cover during a complete turn of the tube. This is made by turning at the joints J_1 and J_2, Fig. 6. The next adjustment is to produce a well defined spot on the cover for it is usually a drawn out band 2 or 3 centimeters in length in a plane perpendicular to the plane of the electrostatic field plates. The beam is not narrow in passing between the plates but spreads out and only a bluish glow is visible. Beyond the plates, however, two converging beams are seen and these generally cross at the lower end of the tube. The appearance is that shown in Fig. 9. It was found

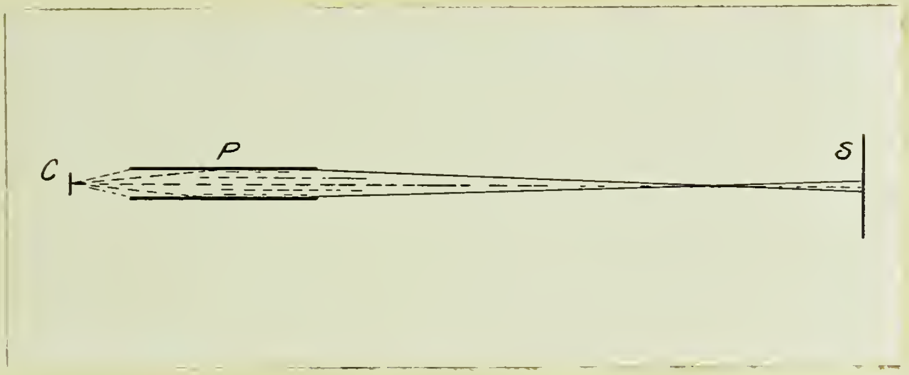

FIG. IX

that the position of crossing depended on the heating current. On varying this it was possible to make the two beams cross right at the screen, or better, a little below so they would cross at the photographic plate. It is only then that a sharp trace and definite spots are found on the plate. There is more than one value of the heating current or temperature at which this adjustment can be made. It is best to take the lower temperature for the number of electrons striking the plate is then enormously less and the film will not be in danger of over exposure while stopping from time to time to apply the potential difference to the electrostatic plates. Having adjusted the beam to sharpness and having waited long enough so conditions are steady, the tube is turned around again to make sure that the beam

[illegible]

[illegible]

[illegible]

will not go off the plate before making the exposure. In making the exposure the commutator is first set going and then the key K (Fig.6) raised to short the electrostatic plates. Noting the position of the tube the shutter is opened and the discharge tube turned slowly through a small angle. The key K in the commutator circuit is then lowered and the beam is allowed to oscillate for 8 or 10 seconds. The key is now raised and the tube is again slowly turned through any desired angle, and an exposure due to the electrostatic deflection is again made. This is continued as often as is desired until the tube has been once around when the shutters are closed immediately. Having admitted air into the tube, the plate holder is removed by gently heating the plate at the bottom of the tube since the half-and-half wax is easily melted. In this connection it should be stated that it was found best to renew the lime for each exposure and since it is practically impossible to get a small new spot over an old one, it is best to renew the platinum strip also.

V RESULTS

Measurements were made on 10 photographic plates, numbered 11-20, the previous 10 plates either lacked the electrostatic deflection because the commutator was not used or were unreliable because a voltmeter of unnecessarily large range was first used whose readings in the neighborhood of 10 volts were found too high. The prints from the plates appear in the appendix. Those from plates 13, 15, 18, 19, 20, are from positives and show the improvement in sharpness of trace and definition of spots as experience was gained in adjusting the beam. The other prints are from the original negatives.

To get the magnetic deflection it is best to measure the diameter of the circle in a direction perpendicular to the lines

joining pairs of spots, for the circle under certain conditions tends towards an elliptical shape. This is especially true if the discharge voltage is much below 800 volts giving the electron a low velocity and hence a large deflection resulting in the lower ends of the plates interfering with the passage of the beam. There can be no such interference in the plane of the plates.

In measuring the electrostatic deflections it is only necessary to take half of the distance between any pair of spots. It should be remarked that the spots appear much more definite on the negative if it is held against a light or a white background than on the prints. Usually 5 measurements were made of the magnetic deflection and 10 of the electrostatic from each plate and mean values used in the calculation of H.

TABLE I

Plate Number	E	V in Volts	z in centimeters	Y in centimeters	H	d
11	0.03504	10.50	2.00	1.75	0.1716	0.0054
12	0.03378	9.63	2.02	1.73	0.1610	0.0052
13	0.03504	9.63	1.98	1.78	0.1614	0.0048
14	0.03504	9.34	1.96	1.59	0.1665	0.0003
15	0.03504	9.33	2.10	1.77	0.1690	0.0028
16	0.03504	10.78	2.13	2.18	0.1660	0.0002
17	0.03504	9.33	2.07	1.66	0.1719	0.0057
18	0.03504	9.33	2.08	1.82	0.1650	0.0012
19	0.03504	9.36	2.205	1.941	0.1662	0.0000
20	0.03504	9.39	2.097	1.88	0.1642	0.0020
				Mean	0.1662	0.0008

In the table it is seen that E is the same for all determinations except No. 12 where it was changed by lowering the

electrostatic field plates. The plates were moved back to that of No. 11 for subsequent measurements. The constants of the tube l and l' were measured with a cathetometer and d and t with a scale, giving

$$l = 46.05 \text{ cm.}$$

$$l' = 42.03 \text{ cm.}$$

$$d = 10.00 \text{ cm.}$$

$$t = 1.52 \text{ cm.}$$

VI CORRECTION FOR IRREGULARITIES OF THE FIELD

In the development of the formula it was assumed that the field between the electrostatic plates was perfectly uniform and did not extend beyond the limits of the plates. This is not the case and a correction must be applied.

In making this correction use is made of Plate XII, Vol. 1, Maxwell's Electricity and Magnetism, where the equipotential surfaces and lines of force at the edges of a parallel plate condenser are given. The field at any point in the median plane can be obtained if we assume that the force is proportional to the density of the lines or inversely proportional to their distance apart. Fig. 10 shows the cathode, electrostatic plates and photographic plates in the positions C, P and S respectively. The distances between the lines of force along the median plane are accurately measured from Maxwell's figure and their reciprccals plotted as ordinates in the curve DEFGH above so they are proportional to the field at points directly below. The height of the rectangle IJKL gives the field at all points below for the ideal case. The ordinates are of course of the same magnitude for both cases for points far removed from the ends of the plates.

The equation of motion is

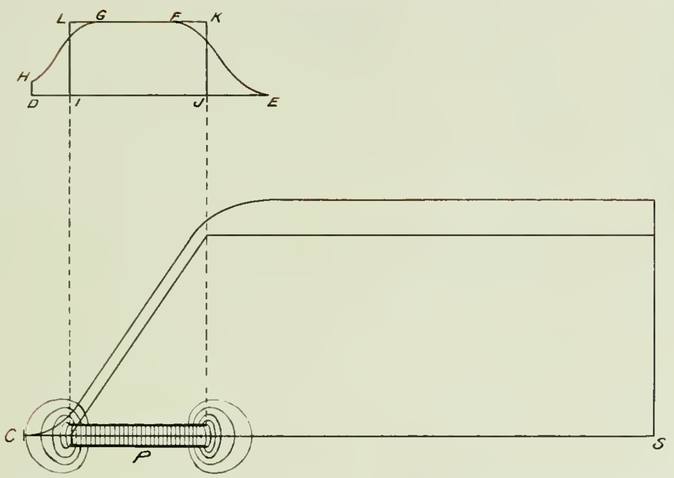

FIG. X

[illegible]
[illegible]
[illegible]

$$\frac{d^2y}{dt^2} = \frac{e}{m}\frac{d\theta}{dy}$$ [1]

where y is measured in a direction perpendicular to the axis of the tube and θ is the potential

A first integral is

$$\frac{dy}{dt} = \frac{e}{mv}\int_0^x \frac{d\theta}{dy}dx$$

since vdt = dx

where x is measured along the axis of the tube.

The integral is

$$y = \frac{e}{mv^2}\int_0^{X_1} dx \int_0^x \frac{d\theta}{dy}dx$$

where X_1 is the distance of the photographic plate from the cathode and $\frac{d\theta}{dy}$ is the electric force and proportional at any point to the ordinates of the curves DEFGH and IJKL so the double integral can be evaluated graphically.

DE is divided into centimeters, say, on a drawing of larger scale than here shown and ordinates drawn at each point of division. The areas from the ordinate DH up to each of the ordinates at division points are then measured with a planimeter and plotted as ordinates for the corresponding values of x in the curve below. In this manner the curve with rounded corners was obtained, it being noticed that the curve becomes horizontal after passing the field because there is no increase in area under the curve DEFGH. The process was repeated for the rectangle IJKL and the inner curve obtained. The areas under these curves are proportional to the electrostatic deflections and their ratio turns out to be 1.166 so the measured deflection is too large in that ratio. The deflection measured on the

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[1] Seitz, Ann. d. Phys., 8, 1902, p. 240.

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

photographic plate must be divided by 1.166 giving for the formula

$$H = Ez\sqrt{\frac{V \times 1.166}{y}}$$

$$= \sqrt{1.166}\ Ez\frac{\sqrt{V}}{y}$$

$$= 1.079\ Ez\frac{\sqrt{V}}{y}$$

The correction factor of 1.079 is applied to the mean value of H given in the table.

Therefore

$$H = 1.079 \times 0.1662 = 0.1793$$

The correction for the probable error of the arithmetical mean given in the table is negligible so

$$H = 0.1793 \pm 0.0008$$

VII DISCUSSION

The distances l and l' which are large and d and t are all conveniently measured so the constant E fixed by them can be determined to the degree of accuracy required by the other data. The voltmeter could be read to 1 part in 1000 but there was fluctuation in the voltage so it is probably not correct to more than 0.3 percent which is also the accuracy to which the factor correcting for the irregularity of the electrostatic field was determined.

Taking Plate No. 19 as an example, none of the measurements of the diameter of the circle giving magnetic deflection differed from the mean of 5 measurements by more than 1 part in 440. The mean deviation of individual measurements of the electrostatic deflection from the mean of 9 measurements was 2 parts in 388, but this enters to the one-half power so that both deflections enter the formula to the same degree of accuracy. The deviations from the mean

given in the last column of the table are large but can be accounted for in part by the presence of electrical circuits in the neighboring rooms of the laboratory. The deviations are consistent with the results always given by Braun tubes, Milham[1] having found that special precautions as to pressure, moisture and time for settling to a steady state were necessary in order that determinations should agree to 1 percent. The agreement among the values of H is as good as that among those given for e/m by cathode rays. The best, as far as agreement among the values is concerned, being those by Seitz[2].

VIII SUMMARY

1 A new method of determining H has been developed which depends on the property of the cathode rays of being deflected by a magnetic and an electrostatic field. The well determined ratio of the charge to the mass of an electron, 1.765×10^7 was assumed.

2 A modified form of Braun tube was used, equipped with the Wehnelt hot lime cathode, adjustable in position, and deflections recorded on a photographic plate.

3 The difficulty of eliminating the earth's magnetic field for a zero reading of deflection was avoided by mounting the tube in a frame free to turn about a vertical axis. A second difficulty in connection with the breaking down of the deflecting electrostatic field owing to ionization of the residual gas was overcome by the use of an alternating electromotive force.

4 A method of correcting for irregularities at the ends of the electrostatic field plates was devised. Use was made of a diagram giving the lines of force and equipotential surfaces near the

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[1] Milham, Phys. Zeit., 1901, p. 637

[2] Seitz., Ann. d. Phys., 8, 1902, pp. 235-243.

edges of a parallel plate condenser and the equation of motion integrated graphically.

5 Ten determinations of H were made and the probable error is in the fourth significant place. The agreement among individual determinations is of the same order as that among the best determinations of the ratio e/m, using cathode tubes.

6 The method is another example of the many interesting and useful applications of cathode tubes and is evidence that it is practicable to use the earth's magnetic field for deflecting purposes when measuring e/m, if that field is known by other means.

The author wishes to acknowledge his indebtedness to Dr. C. T. Knipp to whom the method is due and who gave freely of his time and experimental skill while the work was in progress. Thanks are also due to Professor A. P. Carman for the facilities of the laboratory.

IX APPENDIX

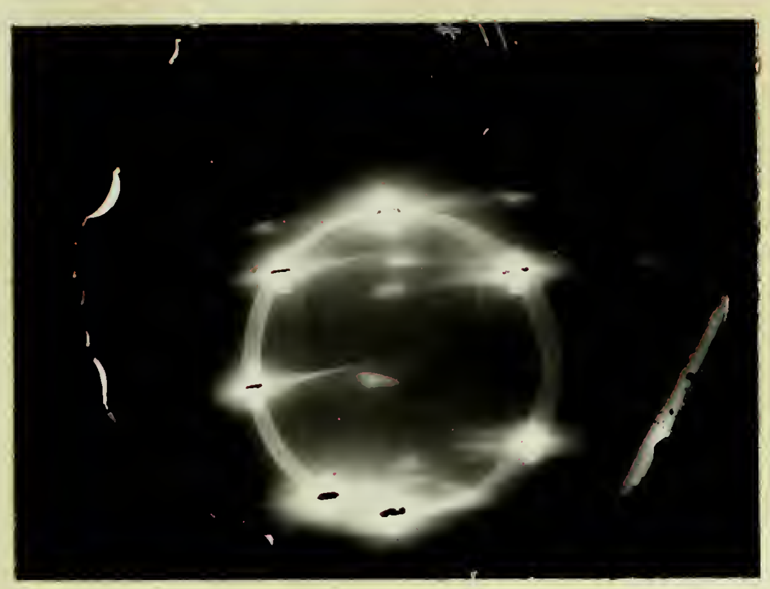

PLATE XI

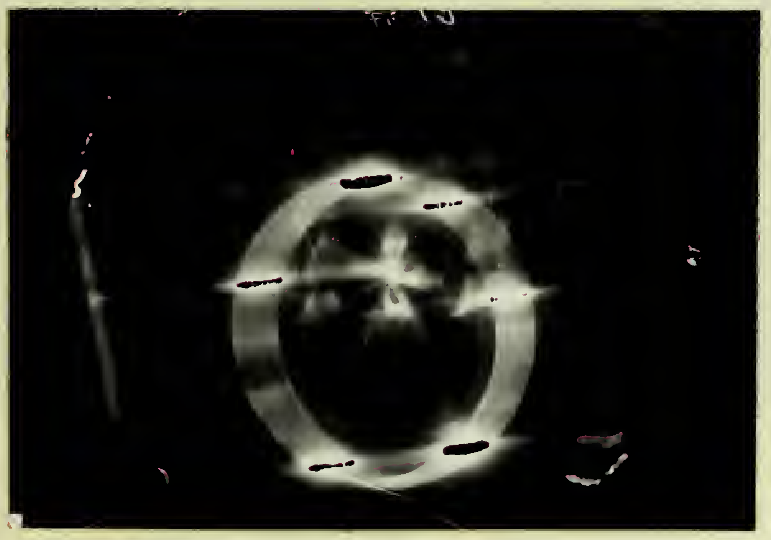

PLATE XII

[illegible]

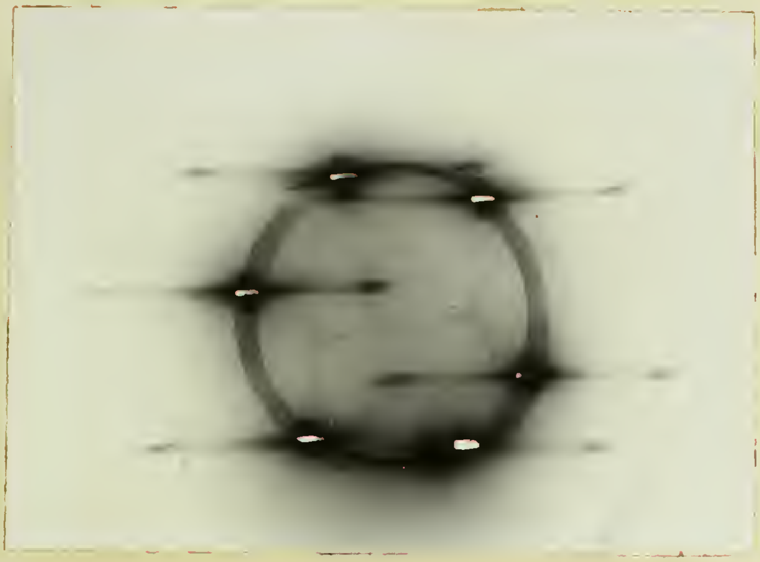

PLATE XIII

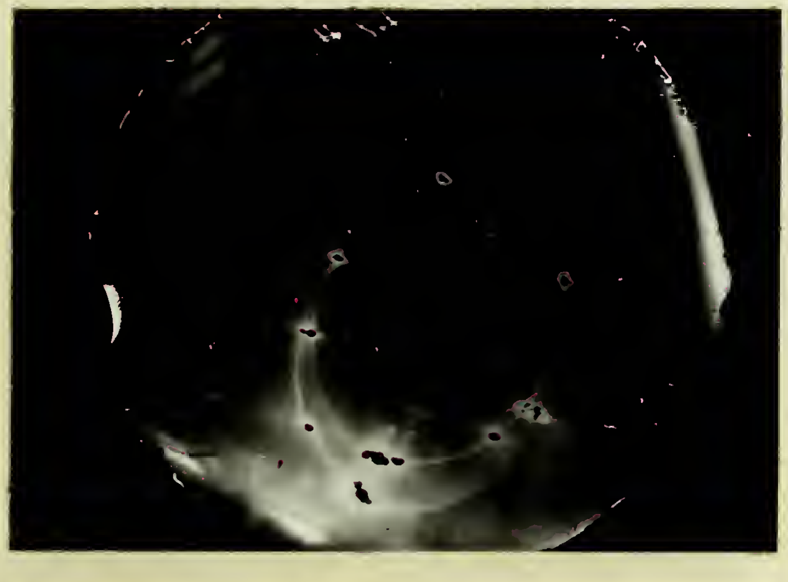

PLATE XIV

[illegible]

[illegible]

[illegible]

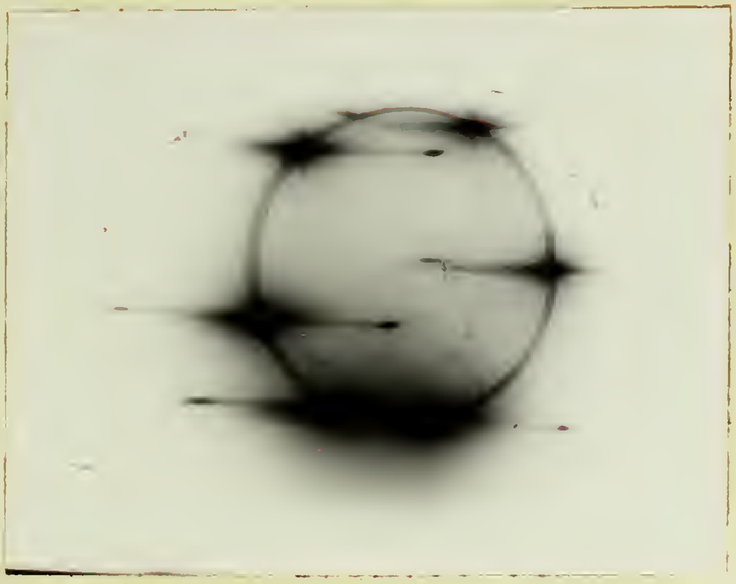

PLATE XV

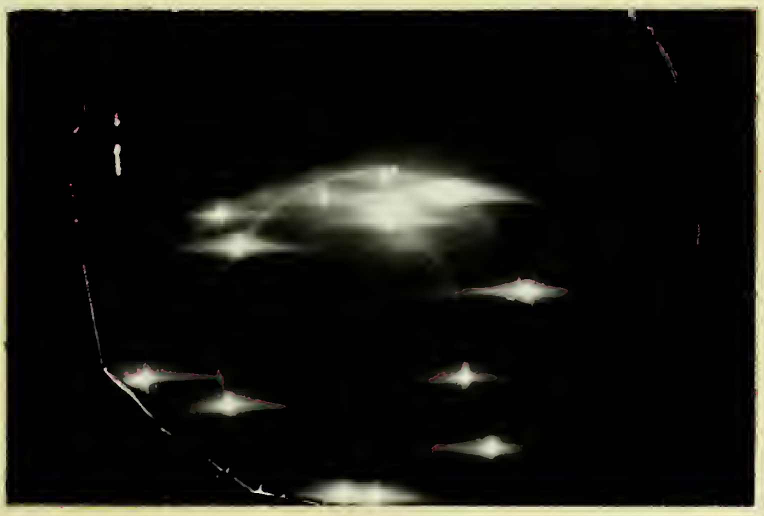

PLATE XVI

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